Pure Mathematics

Algebra and solid geometry concepts

3rd Secondary

Concepts Sheet

Unit one permutations, Combination and Binomial theorem

(1) ${}^{n}P_{r} = n(n-1)(n-2)\dots(n-r+1)\ , n \geq r\ , , n \in Z^{+}$

(2) ${}^{n}P_{r} = \dfrac{\underline{|n}}{\underline{|n-r}}$ **(3)** $\underline{|1} = \underline{|0} = 1$

(4) ${}^{n}C_{r} = \dfrac{{}^{n}P_{r}}{\underline{|r}} = \dfrac{\underline{|n}}{\underline{|r}\ \underline{|(n-r)}}$ **(5)** ${}^{n}C_{n} = {}^{n}C_{0} = 1$

(6) ${}^{n}C_{r} = {}^{n}C_{n-r}$ **(7)** If ${}^{n}C_{X} = {}^{n}C_{Y}$ then $X = Y$ or $X + Y = n$

(8) $\dfrac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \dfrac{n-r+1}{r}$ **(9)** ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

(10) $(X+a)^{n} = X^{n} + {}^{n}C_{1}\, X^{n-1}a + {}^{n}C_{2}\, X^{n-2}a^{2} + \dots + a^{n}$

$(X-a)^{n} = X^{n} - {}^{n}C_{1}\, X^{n-1}a + {}^{n}C_{2}\, X^{n-2}a^{2} - \dots + (-a)^{n}$

(11) $(X+a)^{n} + (X-a)^{n} = 2(\text{Sum of odd ordered terms})$ from $(X+a)^{n}$

(12) $(X+a)^{n} - (X-a)^{n} = 2(\text{Sum of even ordered terms})$ from $(X+a)^{n}$

(13) $(1 \pm X)^{n} = 1 \pm {}^{n}C_{1}\, X + {}^{n}C_{2}\, X^{2} \pm {}^{n}C_{3}\, X^{3} + \dots (\pm X)^{n}$

(14) The general term in the expansion of $(X+a)^{n}$ is $T_{r+1} = {}^{n}C_{r}\, X^{n-r}a^{r}$

The middle term in the expansion $(X+a)^{n}$

(a) If n is odd , there are two middle terms of orders $\frac{n+1}{2}, \frac{n+3}{2}$

(b) If n is even , there is one middle term of order $\frac{n+2}{2}$

(15) In the expansion of $(X+a)^{n}$, The ratio between two consecutive terms

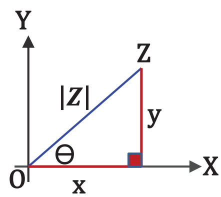
$$\frac{T_{r+1}}{T_{r}} = \frac{n-r+1}{r} \times \frac{a}{X}$$

(16) In the expansion of $(X+a)^{n}$, The ratio between the two coefficients of the two consecutive terms $= \dfrac{n-r+1}{r} \times \dfrac{\text{coefficient of second term}}{\text{coefficient of first term}}$

Unit(2) Complex numbers

- **Complex number** for each $x, y \in R$ thus $Z = x + yi$ is called a complex number whose real part is x and the imaginary part is y where $i^2 = -1$
- **The conjugate of the complex number** If $Z = x + yi$ then its conjugate $\overline{Z} = x - yi$ and $Z + \overline{Z}$ = real number , $Z\overline{Z}$ = real number
- **Properties of the conjugate:**

  (1) $(\overline{Z_1 + Z_2}) = \overline{Z_1} + \overline{Z_2}$ (2) $(\overline{Z_1 Z_2}) = (\overline{Z_1})(\overline{Z_2})$ (3) $\overline{\left(\frac{Z_1}{Z_2}\right)} = \frac{\overline{Z_1}}{\overline{Z_2}}$
- **Geometrical representation of a complex number**: The complex number $Z = x + yi$ is represented by point (x , y) in Argand's plane .
- **The modulus and the amplitude of the complex number**: If point (x , y) represents the complex number Z on Argend's plane, then $|Z| = r = \sqrt{X^2 + Y^2}$ amplitude of Z is got from $cos\theta = \frac{X}{r}$, $sin\theta = \frac{Y}{r}$
- **properties of modulus and amplitude of a complex number**



  (1) $|Z| = |\overline{Z}|$ (2) $Z\overline{Z} = |Z|^2 = |\overline{Z}|^2$

  (3) $|Z_1 Z_2| = |Z_1||Z_2|$ (4) $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$

  (5) $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

  (6) The amplitude of a complex number can take an infinite number of values that each differ by amplitude of 2π

  (7) The amplitude which belongs to the interval $]-\pi, \pi]$ is called the Principle amplitude of a complex number .

  (8) $\arg(\overline{Z}) = -\arg Z$ (9) $\arg(-Z) = -\pi + argZ$

  (10) $arg\frac{1}{Z} = -argZ$

- **The trigonometric form of a complex number:**

  $Z = r(cos\theta + isin\theta)$ where $r = |Z|$ and θ is the principle amplitude

- **Multiplying and dividing complex numbers in a trigonometric form**

If $Z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, $Z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ then

- $Z_1 Z_2 = r_1 r_2\big(cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)\big)$
- $\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\big(cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)\big)$

The exponential form of the complex number (Euler's form) if Z is a complex number whose modulus is r and principle amplitude is θ , then
$Z = re^{\theta i}$ where in radian measure.

$e^{\theta i} = \cos\theta + i\sin\theta$, $e^{-\theta i} = \cos\theta - i\sin\theta$

❖ De Moivre's theorem: If n is a positive real number, then: :

(1) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(2) If K is (+ve) , then $(\cos\theta + i\sin\theta)^{\frac{1}{K}} = \cos\frac{\theta+2n\pi}{K} + i\sin\frac{\theta+2n\pi}{K}$

Thus $(\cos\theta + i\sin\theta)^{\frac{1}{K}}$ takes different values according to n and the number of these different values equals K values which we get by putting r = ... , - 2 , - 1 , 0 , 1 , 2 ... that makes the amplitude $\frac{\theta+2n\pi}{K}$ included between $-\pi , \pi$

❖ The cubic roots of unity: If $Z^3 = 1$ then $Z = 1 , -\frac{1}{2} + \frac{\sqrt{3}}{2}i , -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

And these roots can denoted by 1 , ω , ω^2

❖ Properties of the cubic roots of one:

(1) $\omega^3 = 1$ (2) $1 + \omega + \omega^2 = 0$ (3) $\omega^2 - \omega = \pm\sqrt{3}i$

❖ Properties of the cubic roots of one: If $Z^n = 1$

Then $Z = (\cos 0^\circ + i\sin 0^\circ)^{\frac{1}{n}} = \cos\frac{2\pi K}{n} + i\sin\frac{2\pi K}{n}$, where $K \in Z, \frac{2\pi K}{n} \in]-\pi, \pi]$

The nth roots of one is represented in the Argand's plane by a regular polygon with n vertices which lie on a circle whose center is origin point and radius length equals 1

$\omega = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$, $\omega^2 = -\frac{1}{2} \mp \frac{\sqrt{3}}{2}i$

Unit(3) Determinants and Matrices

Properties of determinants

- In any determinant if the rows are replaced by the columns and the columns are Replaced by the rows in the same order, then the value of the determinant is unchanged.
- The value of a determinant does not change by evaluating it in terms of the elements of any of its rows (columns).
- If there is a common factor in all the elements of any row (column) in a determinant, then this factor can be taken outside the determinant.
- The value of the determinant is equal to zero in each of the following cases:
- If all the elements of any row (column) in a determinant are zeros , then the value of the determinant is zero.
- If the corresponding elements in two rows (columns) of any determinant are equal , then the value of the determinant is zero.
- If the positions of two rows (columns) are interchanged , then the value of the resulted determinant is equal to the value of the original determinant multiplies by (-1) .
- if all the elements of any row (column) are written as the sum of two elements, then the value of the determinant can be written as the sum of two determinants .
- If we add to all the elements of any row (column) a multiple of the elements of another row (column), the value of the determinant is unchanged.
- The value of the determinant in the triangular form is equal to the product of the elements of its main diagonal.

❖ To find the inverse of a 3×3 square matrix , we follow the next steps:

- Find the determinant of the matrix A where $|A| \neq 0$
- Form the cofactors matrix (C) of elements of the matrix A
- Find the adjoint matrix of A (the transpose of the cofactors matrix)
- Find the multiplicative inverse of the matrix using the relation $A^{-1} = \frac{1}{|A|} \times \mathrm{Adj}(A)$

❖ Solving a system of linear equation

Considering A is the coefficients matrix , X is the variable matrix , B is the constants matrix then

- The matrix equation is written in the form $AX = B$
- The solution of this equation if : $X = A^{-1} \times B$

❖ The rank of the matrix:

The rank of the non-zero matrix is the greatest order of determinant or minor determinant of the matrix whose value does not vanish, so if A is a non-zero matrix of the order $m \times n$ where $m \geq n$, then the rank of the matrix A is denoted by $1 \leq RK(A) \leq n$

❖ **The augmented matrix:** It is an extended matrix for a linear system and denoted by A*
Where $A^* = (A|B)$ is of the order $m \times (n+1)$

❖ Non-homogeneous equations:

The system of equations in the form of matrix equation : $AX = B$ is said to be non- homogeneous where $B \neq \boxed{}$

the system of (n) equations in (n) variables has a unique solution if $RK(A) = RK(A^*) = n$, $|A| \neq 0$

- The system has infinite number of solutions if $RK(A) = RK(A^*) = k$ Where $K < n$
- The system has no solution if $RK(A) \neq RK(A^*)$

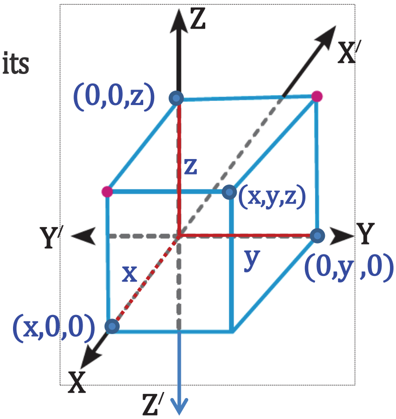
❖ Homogeneous equations:

The system of equations in the form : $AX = \boxed{}$ are called homogeneous equations and If : $RK(A) = RK(A^*) = n$ (number of variables), then the system has a unique solution which is the zero solution (trivial solution)

$RK(A) < n$ (number of variables) , $|A| = 0$, then the system has infinite number of solutions other than the zero solution .

The 3D –Orthogonal Coordinate system:

Identifying the coordinates of point A in the space by knowing its projection on each of the coordinate axes



❖ Cartesian planes

- The Cartesian plane xy its equation is $Z = 0$
- The Cartesian plane xz its equation is $y = 0$
- The Cartesian plane yz its equation is $x = 0$

❖ The distance between two points

If $A(X_1, \ Y_1, \ Z_1)$, $B(X_2, \ Y_2, \ Z_2)$ are two points in the space then
The distance between A and B is given by the relation

$$AB = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2 + (Z_1 - Z_2)^2}$$

❖ The coordinates of the midpoint of a line segment

If $A(X_1, \ Y_1, \ Z_1)$, $B(X_2, \ Y_2, \ Z_2)$ are two points in the space then
The coordinates of point C (mid point of $\overline{AB}$) $\left(\frac{X_1+X_2}{2}, \frac{Y_1+Y_2}{2}, \frac{Z_1+Z_2}{2}\right)$

❖ The equation of the sphere in space

- Standard form of equation of a sphere of radius (r) and center (L,K,N)
 is $(X - L)^2 + (Y - K)^2 + (Z - N)^2 = r^2$
- The equation of the sphere whose center is origin and radius (r)
 is $X^2 + Y^2 + Z^2 = r^2$
- The general form of equation of a sphere of radius (r) and
 $X^2 + Y^2 + Z^2 + 2LX + 2KY + 2NZ + d = 0$

then Centre = (-L,-K,-N) & Radius = $\sqrt{L^2+K^2+N^2-d}$ where $L^2+K^2+N^2 > d$

❖ The position vector in space

If A (A_X, A_Y, A_Z) is a point in space, then the position vector point A with respect to the origin point is $\vec{A} = (A_X, A_Y, A_Z)$

- A_X is called the component of the vector $\vec{A}$ in the direction of x-axis
- A_Y is called the component of the vector $\vec{A}$ in the direction of Y-axis
- A_Z is called the component of the vector $\vec{A}$ in the direction of Z-axis

❖ The norm of a vector

If $\vec{A} = (A_X, A_Y, A_Z)$ its norm $\|\vec{A}\| = \sqrt{(A_X)^2 + (A_Y)^2 + (A_Z)^2}$

❖ Adding and subtracting vectors in space

$\vec{A} = (A_X, A_Y, A_Z)$, $\vec{B} = (B_X, B_Y, B_Z)$ Then

(1) $\vec{A} + \vec{B} = (A_X + B_X\ , A_Y + B_Y\ , A_Z + B_Z)$

(2) $\vec{A} - \vec{B} = (A_X - B_X\ , A_Y - B_Y\ , A_Z - B_Z)$

❖ Multiplying a vector by a real number

If $\vec{A} = (A_X, A_Y, A_Z)$, $K \in R$ then $K\,\vec{A} = (KA_X, KA_Y, KA_Z)$

❖ Equality of vectors in space

$(A_X, A_Y, A_Z) = (B_X, B_Y, B_Z)$ then $A_X = B_X$, $A_Y = B_Y$, $A_Z = B_Z$

❖ The fundamental unit vectors

- $\hat{\imath} = (1, 0, 0)$ The unit vector in the +ve direction of x-axis
- $\hat{\jmath} = (0, 1, 0)$ The unit vector in the +ve direction of y-axis
- $\hat{K} = (0, 0, 1)$ The unit vector in the +ve direction of z-axis

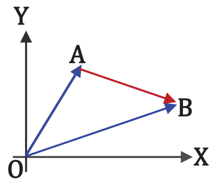
❖ Expressing a vector in terms of the fundamental unit vectors

If $\vec{A} = (A_X, A_Y, A_Z)$ then we can write the vector $\vec{A}$ in the form of $\vec{A} = A_X\vec{\imath} + A_Y\vec{\jmath} + A_Z\vec{k}$

❖ expressing the directed line segment in space in terms of the coordinates of its terminals

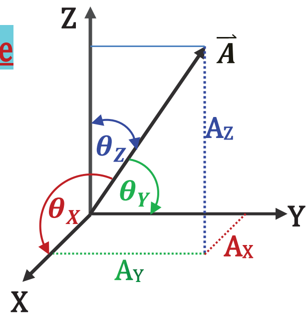
if A and B are two points in space their position vectors are $\vec{A}$ and $\vec{B}$ respectively,

then $\overrightarrow{AB} = \vec{B} - \vec{A}$



❖ The unit vector in the direction of a given vector

If $\vec{A} = (A_X, A_Y, A_Z)$ then ,the unit vector in the direction of $\vec{A}$ **is** $\overrightarrow{U_A} = \frac{\vec{A}}{\|\vec{A}\|}$

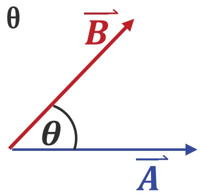


❖ Direction angles and direction cosine of a vector in space

If $\theta_X, \theta_Y, \theta_Z$ are the angles which the vector $\vec{A}=(A_X, A_Y, A_Z)$ with +ve directions of x , y , z axes respectively,

- $A_X = \|\vec{A}\|\cos\theta_X$, $A_Y = \|\vec{A}\|\cos\theta_Y$, $A_Z = \|\vec{A}\|\cos\theta_Z$
- $(\theta_X, \theta_Y, \theta_Z)$ is direction angles of the vector $\vec{A}$
- $(\cos\theta_X, \cos\theta_Y, \cos\theta_Z)$ is called direction cosines of vector $\vec{A}$
- $\cos\theta_X\vec{i} + \cos\theta_Y\vec{j} + \cos\theta_Z\vec{K}$ represent unit vector in direction of $\vec{A}$
- $(\cos\theta_x)^2 + (\cos\theta_Y)^2 + (\cos\theta_Z)^2 = 1$

❖ The scalar product of two vectors:



If $\vec{A}$ and $\vec{B}$ are two vectors in R^3 and the measure of the angle between them is θ where $0 \le \theta \le 180^\circ$, then $\vec{A} \bullet \vec{B} = \|\vec{A}\|\|\vec{B}\|\cos\theta$

❖ The properties of the scalar product of two vectors

(1) $\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$ Commutative properties

(2) $\vec{A} \bullet (\vec{B} + \vec{C}) = \vec{A} \bullet \vec{B} + \vec{A} \bullet \vec{C}$ Distributive properties

(3) If k is a real number, then $(K\vec{A}) \bullet \vec{B} = \vec{A} \bullet (K\vec{B}) = K(\vec{A} \bullet \vec{B})$

(4) $\vec{A} \bullet \vec{A} = \|\vec{A}\|^2$

(5) If $\vec{A} \bullet \vec{B} = 0$ if and only if $\vec{A}$, $\vec{B}$ are perpendicular

❖ The scalar product of two vectors in an orthogonal coordinate system

If $\vec{A} = (A_X, A_Y, A_Z)$, $\vec{B} = (B_X, B_Y, B_Z)$ then $\vec{A} \bullet \vec{B} = A_X B_X + A_Y B_Y + A_Z B_Z$

❖ The angle between two vectors

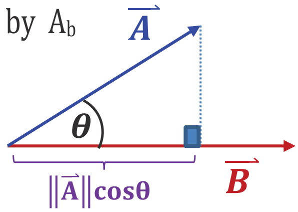
The angle between the two vectors $\cos\theta = \dfrac{\vec{A} \bullet \vec{B}}{\|\vec{A}\| \times \|\vec{B}\|}$

- If $\cos\theta = 1$, then $\vec{A} // \vec{B}$ and on the same direction
- If $\cos\theta = -1$, then $\vec{A} // \vec{B}$ and on the opposite direction
- If $\cos\theta = 0$ then $\vec{A} \perp \vec{B}$

❖ The component of a vector in the direction of another vector

The component of vector $\vec{A}$ in the direction of $\vec{B}$ is denoted by A_b

$$A_b=\|\vec{A}\|\cos\theta = \frac{\vec{A}\bullet\vec{B}}{\|\vec{B}\|}$$



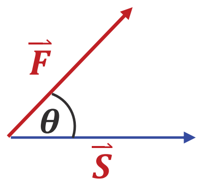
❖ The vector component of $\vec{A}$ in direction of $\vec{B}$

$$= \left(\frac{\vec{A}\bullet\vec{B}}{\|\vec{B}\|^2}\right)\vec{B}$$

❖ The work done by the force $\vec{F}$ to make a displacement $\vec{S}$

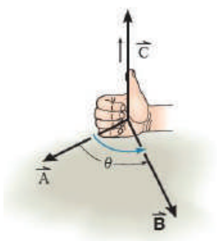
$$\text{The work} = \vec{F}\bullet\vec{S} = \|\vec{F}\|\|\vec{S}\|\cos\theta$$

- If the force $\vec{F}$ is in the direction of the displacement$(\theta = 0^\circ)$, then W = $\|\vec{F}\|\,\|\vec{S}\|$
- If the force $\vec{F}$ is in the opposite direction of the displacement$(\theta = 180^\circ)$, then W = $-\|\vec{F}\|\,\|\vec{S}\|$
- If the force $\vec{F}$ is perpendicular to the direction of the displacement, then W = 0



❖ The vector product of two vectors

If $\vec{A}$ and $\vec{B}$ are vectors in R^3 and the measure of the smallest angle between them is θ, then $\vec{A}\times\vec{B}=(\|\vec{A}\|\|\vec{B}\|\sin\theta)\,\vec{C}$ where $\vec{C}$ is perpendicular unit vector to the plane of $\vec{A}$ and $\vec{B}$. The direction of $\vec{C}$ is identified (up or down) According to the right hand rule where the curved fingers of the right hand to the direction of rotation from $\vec{A}$ to $\vec{B}$ and the thump shows the direction of $\vec{C}$



❖ The properties of the vector product of two vectors

(1) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

(2) $\vec{A}\times\vec{A} = \vec{0}$

(3) $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ distributive property

(4) If $\vec{A}\times\vec{B} = \vec{0}$ then $\vec{A}\,//\,\vec{B}$ or one of the two vectors or both of them equals $\vec{O}$

(5) $\hat{\imath}\times\hat{\jmath} = \widehat{K}$ $\quad \hat{\jmath}\times\widehat{K} = \hat{\imath}$ $\quad \widehat{K}\times\hat{\imath} = \hat{\jmath}$

❖ The vector product of two vectors in a perpendicular coordinates system

If $\vec{A} = (A_X, A_Y, A_Z)$, $\vec{B} = (B_X, B_Y, B_Z)$ then

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

❖ Special case : The vector product in the xy-plane

If $\vec{A} = (A_X, A_Y)$, $\vec{B} = (B_X, B_Y)$ then $\vec{A}\times\vec{B} = \begin{vmatrix} A_X & A_Y \\ B_X & B_Y \end{vmatrix}\vec{K} = (A_X B_Y - A_Y B_X)\vec{K}$

❖ The perpendicular unit vector on the plane of the vectors $\vec{A}$, $\vec{B}$

$$\vec{C} = \frac{\vec{A}\times\vec{B}}{\left\|\vec{A}\times\vec{B}\right\|}$$

❖ Parallelism of two vectors

The two vectors $\vec{A} = (A_X, A_Y, A_Z), \vec{B} = (B_X, B_Y, B_Z)$ are parallel

Then(1) $\vec{A}\times\vec{B} = \vec{0}$ or (2) $\vec{A} = K\vec{B}$ or(3) $\frac{A_X}{B_X} = \frac{A_Y}{B_Y} = \frac{A_Z}{B_Z}$

❖ The geometrical meaning of vector product

$\left\|\vec{A}\times\vec{B}\right\|$ = the area of the parallelogram where $\vec{A}$ and $\vec{B}$ are two adjacent sides

=double the area of triangle where $\vec{A}$ and $\vec{B}$ two adjacent sides

❖ The geometrical meaning of the scalar triple product

the volume of parallelepiped where $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three vectors represent the non parallel edges equals to the absolute value of

$$\vec{A}\bullet\vec{B}\times\vec{C} = \begin{vmatrix} A_X & A_Y & A_Z \\ B_X & B_Y & B_Z \\ C_X & C_Y & C_Z \end{vmatrix}$$

Unit(2) straight lines and planes in the space

❖ Direction vector:

(1)If L, m and n are the direction cosines of a straight line then the vector $\vec{d} = t(L, m, n)$ Represents the direction vector of the straight line and is denoted by $\vec{d} = (a, b, c)$

Where (a, b, c) re called the direction ratios of the straight line

(2) The direction vector of the straight line takes different equivalent forms such as

$\vec{d} = 2(L, m, n) = 3(L, m, n) = -(L, m, n) = \cdots$

❖ Equation of the straight line

The equation of the straight line which passes through point (x_1, y_1, z_1) and the vector $\vec{d} = (a, b, c)$ is its direction vector

- Vector form: $\vec{r} = (X_1, Y_1, Z_1) + t\,(a, b, c)$
- parametric form: $\boxed{X = X_1 + at}$, $\boxed{Y = Y_1 + bt}$, $\boxed{Z = Z_1 + ct}$
- Cartesian form: $\frac{X-X_1}{a} = \frac{Y-Y_1}{b} = \frac{Z-Z_1}{c}$

❖ The angle between two straight lines

If L_1 , L_2 are two straight lines in space whose direction vectors are $\vec{d_1} = (a_1 , b_1 , c_1)$ and $\vec{d_2} = (a_2 , b_2 , c_2)$, then the smallest angle between the two straight lines L_1, L_2 is θ

$$\boxed{\cos\theta = \frac{|\vec{d_1} \bullet \vec{d_2}|}{\|\vec{d_1}\| \times \|\vec{d_2}\|}}$$

and if (L_1 , m_1 , n_1), (L_2 , m_2 , n_2) are the direction cosines for the two straight lines, then: $\cos\theta = |L_1 L_2 + m_1 m_2 + n_1 n_2|$

❖ The parallelism and perpendicularity conditions of two straight lines

The two straight lines are parallel if

If $\vec{d_1} = (a_1, b_1, c_1)$, $\vec{d_2} = (a_2, b_2, c_2)$ are the direction vectors of the two straight lines L_1 , L_2 The two lines area parallel

$\vec{d_1} = K\vec{d_2}$ or $\vec{d_1} \times \vec{d_2} = \vec{0}$ or $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

❖ The equation of the plane

The equation of the plane passing through point (X_1, Y_1, Z_1) and the vector $\vec{n} = (a, b, c)$ is perpendicular to the plane

- Vector form $\vec{n} \bullet \vec{r} = \vec{n} \bullet (X_1, Y_1, Z_1)$
- Standard form $a(X - X_1) + b(Y - Y_1) + c(Z - Z_1) = 0$
- General form $aX + bY + cZ + d = 0$, $d = -ax_1 - by_1 - cz_1$

❖ Angle between two planes

If $\overrightarrow{n_1} = (a_1, b_1, c_1)$, $\overrightarrow{n_2} = (a_2, b_2, c_2)$ are the normal to the plane

Then measure of the angle between the two planes is given by the relation

$$\cos\theta = \frac{|\overrightarrow{n_1} \bullet \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\|\|\overrightarrow{n_2}\|} \quad \text{where } 0^\circ \le \theta \le 90^\circ$$

❖ Parallel and orthogonal planes

If $\overrightarrow{n_1} = (a_1, b_1, c_1)$, $\overrightarrow{n_2} = (a_2, b_2, c_2)$ are the perpendicular vectors to the two planes, then the

- condition of parallelism of the two planes is $\overrightarrow{n_1} // \overrightarrow{n_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- condition of perpendicularity of the two planes is $\overrightarrow{n_1} \bullet \overrightarrow{n_2} = 0$ or $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

❖ The perpendicular length drawn from a point to a plane

The length of the perpendicular drawn from the point $A(X_1, Y_1, Z_1)$ To the plane passes through $B(X_2, Y_2, Z_2)$ and vector $\overrightarrow{n_1} = (a, b, c)$ is perpendicular to the plane whose equation : $aX + bY + cZ + d = 0$

- Vector form $L = \frac{|\overrightarrow{BA} \bullet \vec{n}|}{\|\vec{n}\|}$
- Cartesian form $L = \frac{|aX_1 + bY_1 + cZ_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

❖ Equation of the plane using the intercepted parts from the coordinate axes

If a plane cuts the coordinates axes at points $A(x_1, 0, 0)$, $B(0, y_1, 0)$ $C(0, 0, z_1)$, then the equation of the plane is in the form

$$\frac{X}{X_1} + \frac{Y}{Y_1} + \frac{Z}{Z_1} = 1$$